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Characterization of Sea Clutter Amplitude and Doppler Bin PDFs

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14. ABSTRACT
The document establishes a minimum baseline for reporting the amplitude characterization of sea clutter and its associated parameters. The document mainly concentrates on distributions with two parameters where the two parameters are generally called the shape and scale parameters. Matlab code is provided for method of moment and maximum likelihood fits of common sea clutter distributions.

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Characterization of Sea Clutter Amplitude and Doppler bin PDFs

Contents

1.Background

The distribution of the non-coherent and coherent data from the high grazing angle sea clutter is an important quantity for several reasons including: 1) The general characterization of the sea clutter distribution for simulation and modeling, 2) Various other Signal Processing algorithms (e.g. CFAR) can be optimized by knowing the amplitude distribution of sea clutter.

The document establishes a minimum baseline for reporting the amplitude characterization of sea clutter and its associated parameters. It follows many of the similar data tables that are reported in [1]. The document mainly concentrates on distributions with two parameters where the two parameters are generally called the *shape* and *scale* parameters. Combinations of distributions beyond two parameters such as *mixture distributions* (e.g. K-K, Weibull-Weibull)[2] [6] and non-mixture distributions beyond two parameters (e.g. KA distribution)[2][3] are also mentioned. Some of these are tabulated in Table 3-2 and Table 3-3 of the Appendix. It should be noted that not all of these distributions can necessarily be tied to a phenomenological model.

Matlab code is also provided to estimate the parameters of the distributions listed in this report with the exception of the K-A and K-K models.

1.1 Distributions for Sea Clutter Amplitude Characterization

A survey of the literature shows that the following distributions have been used most often for low grazing angle sea clutter [1][2]:

a) K-Distribution (and its variant K+Noise)
b) Log-Normal
c) Weibull
d) Exponential (Intensity form/Envelope^2) - Rayleigh (Envelope form)

There are currently very few reports in the open literature for the amplitude distribution of high grazing angle data. However beyond the above distributions, the Pareto distribution has been mentioned in a series of recent papers as fitting experimental high grazing angle data [4]. These four mentioned distributions (in their 1-D form) along with their first two moments are mentioned in Table 1. It should be noted that the K-Distribution and Pareto Distribution follow the two scale model of assuming the speckle is Rayleigh with an underlying mean of Gamma (for K Distribution) and inverse Gamma (for Pareto).

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Besides correct amplitude fit, SET-185 panel members should assess if any other distributions that they may propose (beyond these) can readily handle spatial and temporal correlations in their multi-dimensional form. Tables 2 and 3 of the Appendix provide the functional form or provide references for the K+noise, K-K and K-A distributions.

2. Minimum Baseline

The following section establishes a minimum baseline that can be used to report and compare amplitude characteristics of sea clutter. Beyond this minimum baseline, usage of other distributions and techniques to verify accuracy are welcomed (see Section 3). For the minimum baseline, staring mode and intensity (power) are to be used. In the staring mode, care should be taken so that the range-cells that are processed have their maximum difference of grazing angle be less than 0.1 degrees. The following parameters shall be reported:

2.1 Input parameters to report (general and minimum baseline)

- Whether the amplitude Intensity (envelope squared or "power") or amplitude envelope is used
    - Note for minimum baseline **intensity is to be used**.
- If the data is calibrated RCS or just relative power.
- Number of samples used
- Clutter to Noise Ratio (based on the Noise Figure or other calculations)
- Radar range resolution (and type of Waveform – e.g. Chirp or CW or Pulse -- if possible)
- Cross range resolution
- PRF (or CW)
- Radar carrier frequency
- Polarization
- Grazing angle
- Wind direction
- Swell Direction
- Wave Height
- Estimated Sea State (Beaufort or Douglas)
- Geographical area
- Other related sea and weather parameters available (e.g. surface temperature, ducting, etc.)

2.2 Minimum Baseline Processing

The following parameters and processing should be used for the minimum baseline

- 5% degrees variability for the grazing angle
- 15% degree variability for scanning mode.
- Intensity (power) rather than amplitude (voltage) Envelope
- Report CNR
- Ideally all available samples will be used for processing as the more samples are used the lower the false alarm region that can be calculated. However this can be limited by not having enough samples or not enough computing power to handle all available samples. Consequently as per 2.1, it is very important to report the number of samples used for each estimate.
- For K-Distribution, Weibull, Log-Normal , Exponential fits:
    - Use of the first two Moments for the fit [1] (see also Appendix)
- K+Noise
    - See [5] (9.46]
- Pareto, (in the addition to the above):
    - Maximum likelihood estimate (MLE) (Appendix).

2.3 Output Parameters (Minimum Base-line)

Input parameter (see 2.1) shall be tabulated and the following output shall be generated:

- Tabulated results where the shape γ and mean parameters (see Appendix – Table 1) are reported for each of the methods mentioned in Section 2.2.
- One Weibull CDF Plots from 0.5 up to a false alarm rate of one in a million.
    - Plot of actual CDF, K1, KN, Log Normal, Weibull, Exponential and Pareto using with the y-axis topping off at (1-1e-6).
- Validation test for the moments (K1, KN, Log-Normal, Rayleigh, Weibull,Pareto) using the ratio of theoretical to observed moments (up to the sixth moment [6])
- Tabulated results of the Modified Chi-Square test for all distributions (see appendix)
- Maximum Deviation (Psuedo K-S Test)

3. Suggestions Beyond the Minimum Base-line

3.1 Doppler Bin Distribution

Similar to the amplitude, the distribution at a frequency bin is of interest for coherent clutter generation. One can take FFT of varying lengths over a range cell time-series as explained in [5]. However this is optional data reporting t is beyond the minimum baseline, but highly encouraged. The method of spectrum estimation (e.g. window, overlap and size of samples) should be carefully reported.

3.2 Other Distributions

Distributions that take noise into account as well as distributions with more than two parameters have also been proposed. Some of these are mentioned in Table 2 and Table 3 in the Appendix. Note these are just a small example of distributions in the literature and results for any other distribution beyond the minimum baseline and the table (e.g. Pareto+Noise) is encouraged.

4. Appendix

4.1 Method of Moments

The method of moments (MM) and the maximum likelihood estimate (MLE) are two common methods for assessing the distribution fit. The MM for each of the four two parameter distributions is given in Table I. Here γ is the shape factor and μ is the scale factor.

The MM estimate of the nth moment of the intensity (z(i) where i is the *ith* sample) is:

$$\boldsymbol{E(r^n) \approx \frac{1}{N_s}\sum_{i=1}^{N_s} z(i)^n}$$

4.2 Maximum Likelihood

The Maximum Likelihood estimate [6] is another estimator for the parameters of sea clutter. It usually requires more sample data than the method of moments for correct convergence. The Weibull and Log-Normal have closed for solutions for the maximum likelihood estimate whereas the Pareto distribution requires a numerical algorithm that is described in [7] and used in Matlab. The following Matlab routines using the statistical toolbox obtain the MLE parameters.

Exponential:
```
parmhat = expfit(data)
```
Log-Normal:
```
parmhat = lognfit(data)
```
Pareto:
```
parmhat = gpfit(X)
```
Weibul:
```
parmhat = wblfit(data)
```

Example:
Generate 100 Weibull sample points with scale parameter 0.5 and shape parameter 0.8
```
data = wblrnd(0.5,0.8,100,1);
```
Use MLE:
```
[parmhat, parmci] = wblfit(data)
parmhat =
  0.5861  0.8567
```

4.3a Fidelity Test 1: Ratio of Theoretical and Observed Moments

From [1,Section 3.2.2 pg 25]: To provide a comparison of the different non-Rayleigh sea clutter models that have been applied to the collected data, the ratio of the theoretical to the observed sample moments for each combination of the parameter estimation and model considered were calculated and compared [1]. See [1] for a clean way of tabulating this result.

4.3b Fidelity Test 2: Modified Chi-square test

See [1, Section 3.2.3 – pg 26 and also the references there in to Chan's work] and also [8] for full description. The modified Chi-Square test can be performed by the following equation:

$$\chi_m^2 = \sum_{i=1}^{K} \frac{\left[f(i) - \frac{N * 0.1}{K}\right]^2}{\frac{N * 0.1}{K}}$$

Where $f(i)$ is the observed number of clutter samples having amplitude within the ith interval, N is the total number of amplitude samples forming the histogram, and (N/K) is the expected number of occurrences in each interval of the statistical model used. "This is equivalent to assigning a zero weighting in the amplitude region where the ***Pfa*** is greater than 0.1 and a uniform weighting in each of the *K* intervals."[7,8] .

4.4 Note on Matlab Program

Matlab program to calculate the parameters for the Weibull, Log-Normal, K, Exponential and Pareto using the maximum likelihood and methods of moments is provided at the end of this report. In addition, the Modified Chi-square Test and Ratio of Theoretical to Observed moments are calculated in the program.

4.4 Distributions Table

Table 1:

Distribution	PDF-P(z)	$E(z^n)$
Log-Normal	$\frac{1}{z\sqrt{2\pi\gamma^2}}\exp\left(-\frac{(\ln(z)-\mu)^2}{2\gamma^2}\right)$	$\exp\left(n\mu+\frac{\gamma^2 n^2}{2}\right)$
K	$\frac{2\mu^{(\gamma+1)/2} z^{(\gamma-1)/2}}{\Gamma(\gamma)} K_{\gamma-1}(2\sqrt{\mu z})$	$n!\frac{\Gamma(\gamma+n)}{\mu^n\Gamma(\gamma)}$
Weibull	$\frac{\gamma}{\mu}\left(\frac{z}{\mu}\right)^{\gamma-1} \exp\left(-\left[\left(\frac{z}{\mu}\right)\right]^{\gamma}\right)$	$\gamma^n\Gamma(1+n/\mu)$
Pareto	$\frac{\gamma\mu^{\gamma}}{(z+\mu)^{\gamma+1}}$ $(See\ Also\ Matlab\ Code)$	$\frac{n!\,\mu^n}{(\gamma-1)(\gamma-2)\ldots(\gamma-n)}$
Exponential	$\left(\frac{1}{\gamma}\right)\exp\left(\frac{-z}{\gamma}\right)$	$n!\,\gamma^n$

Table 2 – Distributions with Noise taken into account

PDF	P(z)	Number of Parameters
K+Noise	$\int_0^{\infty}\frac{\mu^{\gamma} z^{\gamma-1}}{\Gamma(\gamma)}\frac{\exp(\mu x)}{x+P_n}\exp\left(\frac{-z}{x+P_n}\right)dr$	Three (one for noise power P_n) See A.182 [5] for MoM.
KK+Noise	See [6]	Seven (reduced by some parameters being equal)

Table 3 – Distributions with more than two parameters

PDF	P(r)	Number of Parameters
K-K	$\alpha P(r)+(1-\alpha)P(r)$ where P(r) is given above for the K [2] [6]	Five
K-A	See [2] and [3]	Five (reduced by some parameters being equal)

5. Notes

[1] Antipov, Irina. *Statistical analysis of northern Australian coastline sea clutter data*. DSTO Electronics and Surveillance Research Laboratory, 2001. Available from www.scholar.google.com

[2] Dong, Yunhan. *Distribution of X-band high resolution and high grazing angle sea clutter*. No. DSTO-RR-0316. DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION EDINBURGH (AUSTRALIA) ELECTRONIC WARFARE AND RADAR DIVISION, 2006. Available from www.scholar.google.com

[3] Ward, Keith D., Simon Watts, and Robert JA Tough. *Sea clutter: scattering, the K distribution and radar performance*. Vol. 20. IET, 2006.

[4] Weinberg, G. V. "Assessing Pareto fit to high-resolution high-grazing-angle sea clutter." *Electronics letters* 47.8 (2011): 516-517.

[5] Watts, S., "Modeling and Simulation of Coherent Sea Clutter", IEEE transactions on aerospace and electronic systems vol. 48, no. 4 October 2012.

[6] Rosenberg, L., D. J. Crisp, and N. J. Stacy. "Analysis of the KK-distribution with medium grazing angle sea-clutter." *Radar, Sonar & Navigation, IET* 4.2 (2010): 209-222

[7] Hosking, Jonathan RM, and James R. Wallis. "Parameter and quantile estimation for the generalized Pareto distribution." *Technometrics* 29.3 (1987): 339-349.

[8] Chan, Hing C. *Analysis of the north truro sea clutter data*. No. DREO-1051. DEFENCE RESEARCH ESTABLISHMENT OTTAWA (ONTARIO), 1990. Available from www.scholar.google.com

6. Accompanying Matlab Program

```
%%Matlab Program to estimate parameters of the distributions
%%See accompanying word document
%%
%%
%%
clear all
%format LONGENG
format short

%% generate K and K plus noise data
%
speckle=randn(100000,10)+sqrt(-1)*randn(100000,10);
speckle_power=mean(mean((abs(speckle)).^2));
speckle=sqrt(1/speckle_power)*speckle;
% %speckle_power=mean(mean((abs(speckle)).^2))
%
%
scale=1; %%scale= shape/mean_powere, texture power= shape/scale
shape=2;
texture = gamrnd(shape,scale,[100000,10]);
texture_power=mean(mean((abs(texture))));
%
noise_to_clutter_ratio=0.2;
noise=(randn(100000,10)+sqrt(-1)*randn(100000,10));
pn=mean(mean((abs(noise)).^2));
noise=sqrt(noise_to_clutter_ratio*texture_power/pn)*noise;
pn=mean(mean((abs(noise)).^2));
%
%
data=speckle.*sqrt(texture); %for K

data=data+noise; %for K+Noise - comment out for K

% %data_power=mean(mean((abs(data)).^2))
%
% %data_power=mean(mean((abs(data)).^2))
data=data.^2;  %%change from amplitude to power
%
%%



%%
%generate Weibull data
%data=wblrnd(1,2,1,1e6);
%%

%%
%generate lognormal data
```

```
%data=lognrnd(.1,.4,1,1e5);

%%generate Pareto data
%data=gprnd(-.3,1,0,1,1e5);


%%
%generate exponential
%data=exprnd(.5,1,1e5);
%%




%%
%%Reshape Data as a Single Vector and obtain its CDF
data=abs(reshape(data ,1,numel(data)));
EnvData=sqrt(data);
LB=10*log10(min(min(abs(data))));
UP=10*log10(max(max(abs(data))));
z=LB:.01:UP;
z=10.^(z/10);
Fhist=hist(data,z);
Fcdf=cumsum(Fhist./sum(Fhist));
Fcdf(end)=1;
%%

%%
%Modified Chi Square Parameters
K=10; % levels
prob_start=.1; %note chan's test starts with 0.1
cd_prob_start=1-prob_start;
eq_int=cd_prob_start:((1-cd_prob_start)/(K)):1; %equal probability  %
modified Chi-Square test

%%

%%get moms for intensity (proprotional to RCS)%%
mom1=1/(numel(data))*sum(data);
mom2=1/(numel(data))*sum(data.^2);
mom3=1/(numel(data))*sum(data.^3);
mom4=1/(numel(data))*sum(data.^4);
mom5=1/(numel(data))*sum(data.^5);
mom6=1/(numel(data))*sum(data.^6);
moms=[mom1 mom2 mom3 mom4 mom5 mom6];
%%


%%
%Envelope=sqrt(data);
%Emom1=1/(numel(data))*sum(Envelope);
%Emom2=1/(numel(data))*sum(Envelope.^2);
%Emom3=1/(numel(data))*sum(Envelope.^3);
%Emom4=1/(numel(data))*sum(Envelope.^4);
%Emom5=1/(numel(data))*sum(Envelope.^5);
%Emom6=1/(numel(data))*sum(Envelope.^6);
```

```
%Emoms=[Emom1 Emom2 Emom3 Emom4 Emom5 Emom6];
%%

%%Weibull Mom%%
v=0:.0001:10;  % note v=2 is Rayleigh and below two is Spiky
val=abs((1./(gamma(1./v+1).^2)).*gamma(2./v+1)-mom2/(mom1^2));
[a,b]=min(val);
v=v(b);
u=mom1/gamma(1/v+1);

%MLE alternative
%[u_v] = wblfit(data);
%u=u_v(1), v=u_v(2);


%CDF
CDFestimatedW=wblcdf(z,u,v);

%Ratio of theortetical to actual moments
n=1:6;
TM_Weibull=u.^(n).*gamma(n/v+1);   %theoretical moments Weibull
MoMratio_Weibull=[TM_Weibull./moms];

%%Modified Chi Square Test
int = wblinv(eq_int,u,v);
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%WblChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
wblmodChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));


%


%%


%%Lognormal MOM%%

%%MoM
u=(log(mom2)-4*log(mom1))/(-2);
v=sqrt((log(mom2)-2*u)/2);  %sigma..
wblpram=[u v];

%MLE alternative
%[u_v] = lognfit(data);
%u=u_v(1), v=u_v(2);


%CDF
CDFestimatedL=logncdf(z,u,v);
```

```
%Ratio of theoeretical to actual moments
n=1:6;
TM_logn=exp(n*u+n.^2*v^2/2);
MoMratio_logn=[TM_logn./moms];

%%Modified Chi Square Test
int = logninv(eq_int,u,v);
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%lognChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
lognmodChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));


%%
%%Exponential MOM
%%
%method of moments
u=mom1;
expparm=u;
%MLE alternative
%[u] = expfit(data);

CDFestimatedE=expcdf(z,u);
n=1:6;
TM_exp=u.^n.*factorial(n);
MoMratio_exp=[TM_exp./moms];
int = expinv(eq_int,u,v);
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%lognChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
expmodChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));


%
%
%%
%%Pareto Fit %%
P=gpfit(data);


CDFestimatedP = gpcdf(z,P(1),P(2));
int = gpinv(eq_int,P(1),P(2));
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%lognChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
ParmodChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));

% I like this form better for the PDF P(z)=a*b^a/(b+z)^(a+1) but Matlab uses
this form 1/alpha*(1+k*x/alpha)^(-1-1/k)
u=P(2)/P(1);
v=1/P(1);
parparm=[u v];
```

```
%Calculate theoretical ratio of moments - could probably write it as a
%gamma function ratio with negative argument..brute forcing it..
TM_Pareto(1)=factorial(1)*u/(v-1);
TM_Pareto(2)=factorial(2)*(u^2)/((v-1)*(v-2));
TM_Pareto(3)=factorial(3)*(u^3)/((v-1)*(v-2)*(v-3));
TM_Pareto(4)=factorial(4)*(u^4)/((v-1)*(v-2)*(v-3)*(v-4));
TM_Pareto(5)=factorial(5)*(u^5)/((v-1)*(v-2)*(v-3)*(v-4)*(v-5));
TM_Pareto(6)=factorial(6)*(u^6)/((v-1)*(v-2)*(v-3)*(v-4)*(v-5)*(v-6));

MoMratio_Pareto=[TM_Pareto./moms];




%%
%K-1 Distribution MoM
% uses first and second moment of intensity

v=2*mom1^2/(mom2-2*mom1^2);
u=v/mom1;
Kparm=[u v];

CDFestimatedK1=1-(2/gamma(v))*u^(v/2)*((z).^(v/2)).*besselk(v,2*sqrt(z*u));
n=1:6;
TM_K1=factorial(n).*gamma(v+n)./(u.^n*gamma(v));  %4.28a WTW and
[E[z]=[E(env^2)]]
MoMratio_K1=[TM_K1./moms];

t=0:1e-3:z(end);
CDFK1=1-(2/gamma(v))*u^(v/2)*((t).^(v/2)).*besselk(v,2*sqrt(t*u));

for i=1:numel(eq_int)
    [a,b]=min(abs(CDFK1-eq_int(i)));
    int(i)=t(b);
end
int(end)=Inf;
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%lognChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
K1modChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));




%%

%%K+N noise
%% Note for K+N, a subroutine calc_KDF_int is called which is not included.
%% One can remove the K+N portion if they do not have calc_KDF_int
%% Two methods are provided.  One if the noise power is known.  The other
without the noise power.
%Part I - Estimate CDF based on
%method 1 (based on 9.44 and 9.46  we have assumed the moments contain both
%noise and clutter.  And Pn (the noise power) is known.
```

```
%pn=.1;


%v=2*(mom1-pn).^2/(mom2-2*mom1^2);
%u=v/(mom1-pn);
%pc=mom1-pn;
%pc=v/u;

%%
%Part II - Estimate CCDF on Equation .9.49
%method 2 (based on..Equation 9.49)
v=(18*(mom2-2*(mom1^2))^3)/((12*mom1^3-9*mom1*mom2+mom3)^2);
pn=mom1-(v/2*(mom2-2*mom1^2))^(.5);
u=v/(mom1-pn);
pc=mom1-pn;
%%
KNparm=[u v pn];

%% get CDF
clear CDFestimatedKN;
for kk=1:numel(z)
CDFestimatedKN(kk) = calc_KDF_int(z(kk), pc, pn,v, 1, 'cdf');
end

%%

%Part III - Modified Chi Square test
t=0:1e-2:z(end);
clear CDFKN;
for kk=1:numel(t)
CDFKN(kk) = calc_KDF_int(t(kk), pc, pn,v, 1, 'cdf');
end

for i=1:numel(eq_int)
    [a,b]=min(abs(CDFKN-eq_int(i)));
    int(i)=t(b);
end
int(end)=Inf;
[fi]=histc(data,int);
fi(end)=[];
N=numel(data);
%lognChiSqr=sum((fi-N/K).^2/(N/K)); Chi-Square
KNmodChi=sum((fi-(prob_start)*N/K).^2/((prob_start)*N/K));



%pn=5;
%Part IV
%calculate theoretical moments
 TM_KN=zeros(1,6);  %(based on Equation A1.182) alternatively one can use
% the new values of u and v as below
 for n=1:6
     for q=0:n
            if n==q
                valmoms=1;
            else
```

```
            valmoms=moms(n-q);
        end

        TM_KN(n)=TM_KN(n)+(-
1)^q*(pn^q)*valmoms*((factorial(n))^2)*1/(factorial(q)*((factorial(n-q))^2));
    end
 end



MoMratio_KN=[TM_KN./moms];
%%


%% figures  - Method of Moments for everyone but Pareto (MLE)
figure

plot(z,log(log(1./(1-Fcdf))),'b-');
hold on
grid on

%plot(10*log10(z),log(log(1./(1-FCDFestimatedW))),'r:');


plot(z,log(log(1./(1-CDFestimatedW))),'r-');

plot(z,log(log(1./(1-CDFestimatedL))),'g-.');

plot(z,log(log(1./(1-CDFestimatedE))),'k:');


plot(z,log(log(1./(1-CDFestimatedK1))),'b--');

plot(z,log(log(1./(1-CDFestimatedKN))),'m--');

plot(z,log(log(1./(1-CDFestimatedP))),'k-');


legend('CDF','Wbl','Log', 'Exp', 'K1', 'KN', 'Pareto','location','SouthEast')

p     = [0.05 0.10 0.25 0.5,0.75 0.90 0.96 0.99 0.999 0.9999 0.99999
0.999999];

label= str2mat('0.05','0.10','0.25','0.50','0.75','0.90','0.96','0.99',
'0.999','0.9999','0.99999','0.999999');

tick  = log(log(1./(1-p)));

ax=axis;
ax(3)=tick(1);
ax(4)=tick(end);
axis(ax);
```

```
set(gca,'YTickLabel',label)
set(gca,'yTick',tick)
%xlabel('Envelope in dB')
xlabel('\lambda-proportional to RCS','fontsize',12)
ylabel('CDF','fontsize',12)
title('CDF distribution fit')
%%


%% figures  - Method of Moments for everyone but Pareto (MLE) (closer look)
figure

plot(z,log(log(1./(1-Fcdf))),'b-');
hold on
grid on

%plot(10*log10(z),log(log(1./(1-FCDFestimatedW))),'r:');


plot(z,log(log(1./(1-CDFestimatedW))),'r-');

plot(z,log(log(1./(1-CDFestimatedL))),'g-.');

plot(z,log(log(1./(1-CDFestimatedE))),'k:');


plot(z,log(log(1./(1-CDFestimatedK1))),'b--');

plot(z,log(log(1./(1-CDFestimatedKN))),'m--');

plot(z,log(log(1./(1-CDFestimatedP))),'k-');



legend('CDF','Wbl','Log', 'Exp', 'K1', 'KN', 'Pareto','location','SouthEast')

[a,b1]=min(abs(Fcdf-.9));
[a,b2]=min(abs(Fcdf-.9999999));

p     = [0.90 0.96 0.99 0.999 0.9999 0.99999 0.999999];

label= str2mat('0.90','0.96','0.99', '0.999','0.9999','0.99999','0.999999');

tick  = log(log(1./(1-p)));

ax=axis;
ax(1)=z(b1);
ax(2)=z(b2);
ax(3)=tick(1);
ax(4)=tick(end);
axis(ax);
set(gca,'YTickLabel',label)
set(gca,'yTick',tick)
```

```
%xlabel('Envelope in dB')
xlabel('\lambda','fontsize',12)
ylabel('CDF','fontsize',12)
title('CDF distribution fit')
%%

figure
hold on
plot(1:6,MoMratio_Weibull,'r-');
plot(1:6,MoMratio_logn,'g-.');
plot(1:6,MoMratio_exp,'k:');
plot(1:6,MoMratio_K1,'b--');
plot(1:6,MoMratio_KN,'m--')
plot(1:6,MoMratio_Pareto,'k-')

legend('Wbl','Log', 'Exp', 'K1', 'KN','Pareto')
title('Ratio of theoretical to actual moments')
grid on;
ax(1)=1;
ax(2)=6;
ax(3)=-3;
ax(4)=10;
axis(ax);
xlabel('moment number')
ylabel('ratio')
label=str2mat('1','2','3','4','5','6');
tick=1:6;
set(gca,'XTickLabel',label)
set(gca,'XTick',tick)



%%
% Maximum Deviation test below false-alarm of 1e-2
% With independent data from the estimate, it could be used as a K-S test
figure
[a,b]=min(abs(abs(1-Fcdf)-1e-2));
Wbl_KS=max(abs(Fcdf(b:end)-CDFestimatedW(b:end)));
logn_KS=max(abs(Fcdf(b:end)-CDFestimatedL(b:end)));
E_KS=max(abs(Fcdf(b:end)-CDFestimatedE(b:end)));
K1_KS=max(abs(Fcdf(b:end)-CDFestimatedK1(b:end)));
KN_KS=max(abs(Fcdf(b:end)-CDFestimatedKN(b:end)));
P_KS=max(abs(Fcdf(b:end)-CDFestimatedP(b:end)));


MDtest=([Wbl_KS, logn_KS, E_KS, K1_KS, KN_KS, P_KS]);

stem(1:6,MDtest);
ax=axis;
ax(1)=-1;
ax(2)=7;
axis(ax);
hold on
label1=['Wbl'];
label2=['Log'];
label3=['Exp'];
```

```
label4=['K1 '];
label5=['KN '];
label6=['Par'];

label=[label1;label2;label3;label4;label5;label6];
tick=1:6;;
set(gca,'XTickLabel',label)
set(gca,'XTick',tick)
grid on;
xlabel('Distribution Fit')
ylabel('Maximum Deviation in CDF')
title('Maximum Deviation test below a false alarm rate of 1e-2')

%%




%Modified Chi-Square test below false-alarm of 0.1
figure
ModChi_test=([wblmodChi lognmodChi expmodChi K1modChi KNmodChi ParmodChi]);

stem(1:6,ModChi_test);
ax=axis;
ax(1)=0;
ax(2)=7;
ax(3)=0;
ax(4)=1e4;
axis(ax);
hold on
label1=['Wbl'];
label2=['Log'];
label3=['Exp'];
label4=['K1 '];
label5=['KN'];
label6=['Par'];

%label=[label1;label2;label3;label4;label5;label6];
tick=1:6;;
set(gca,'XTickLabel',label)
set(gca,'XTick',tick)
grid on;
xlabel('Distribution Fit')
ylabel('Modified Chi-Square Value')
title('Chi-Square test below a false alarm rate of 0.1')
%%
```